Radiometric Calibration Method Using Interferogram Differencing and Complex Spectra

Ronald C. Carlson
SSAI/NASA Goddard Space Flight Center
Code 693.0
Revised January 17, 2001

I_{Targ} — interferogram recorded when viewing a target of interest at temperature = T_{Targ}

I_{Cold} — interferogram recorded when viewing a cold blackbody calibration target at temperature = T_{Cold}

I_{Warm} — interferogram recorded when viewing a warm blackbody calibration target at temperature = T_{Warm}

υ — frequency in wavenumbers (cm^{-1})

$c_{Targ}(\upsilon)$ — $FFT(I_{Targ})$ = uncalibrated complex spectrum of the target of interest at temperature = T_{Targ}

$c_{Cold}(\upsilon)$ — $FFT(I_{Cold})$ = uncalibrated complex spectrum of the cold blackbody calibration target at temperature = T_{Cold}

$c_{Warm}(\upsilon)$ — $FFT(I_{Warm})$ = uncalibrated complex spectrum of the warm blackbody calibration target at temperature = T_{Warm}

θ — instrument phase = the small angle of rotation of the instrument emission spectrum from 180°

$r(\upsilon)$ — complex instrument spectral response

$R(\upsilon)$ — scalar instrument spectral response

I_{Instr} — interferogram due to instrument emission = the instrument (self-emission) contribution to I_{Targ}

$B_{Instr}(\upsilon)$ — spectral radiance of the instrument (self-emission) in W cm^{-2} $ster^{-1}/cm^{-1}$

$B_{Targ}(\upsilon)$ — spectral radiance of the target of interest in W cm^{-2} $ster^{-1}/cm^{-1}$ at temperature = T_{Targ}

$B_{Cold}(\upsilon)$ — spectral radiance of the cold blackbody calibration target in W cm^{-2} $ster^{-1}/cm^{-1}$ = the Planck function at temperature = T_{Cold}

$B_{Warm}(\upsilon)$ — spectral radiance of the warm blackbody calibration target in W cm^{-2} $ster^{-1}/cm^{-1}$ = the Planck function at temperature = T_{Warm}

T_{Instr} — the temperature of the CIRS instrument ~ 170.0K

I_{Targ}, I_{Cold}, and I_{Warm} are averages of 28 – 30 scans for the ground testing case.

FFTs and inverse FFTs are complex. All FFTs are performed on one or two-sided interferograms, with or without phase correction or apodization. The interferograms do not have to be symmetric about their ZPDs. All calculations are carried out as complex arrays through the final calculation of $B_{Targ}(\upsilon)$ (Equations (5) and (18)). $B_{Targ}(\upsilon)$ is real; its imaginary part consists only of noise.

Focal Plane 1:

The far-infrared Focal Plane 1 is calibrated by viewing only a cold target (deep space at 2.7K). A single calibration target suffices because FP1 is at the same temperature as the instrument = T_{Instr} ~ 170.0K, and we can assume that $B_{Instr}(\upsilon) = B_{170}(\upsilon)$ = the Planck function at temperature = 170.0K. Equations (1a) and (1b) express the linear relationship between the uncalibrated complex spectra and the target radiances:

$$c_{Targ}(\upsilon) = FFT(I_{Targ}) = r(\upsilon)B_{Targ}(\upsilon) + r(\upsilon)B_{Instr}(\upsilon)e^{i\theta} \quad (1a)$$

$$c_{Cold}(\upsilon) = FFT(I_{Cold}) = r(\upsilon)B_{Cold}(\upsilon) + r(\upsilon)B_{Instr}(\upsilon)e^{i\theta} \quad (1b)$$

The instrument phase angle = θ is expected to be 180° for FP1. Therefore, 1a and 1b may be rewritten as:

$$c_{Targ}(\upsilon) = FFT(I_{Targ}) = r(\upsilon)B_{Targ}(\upsilon) - r(\upsilon)B_{Instr}(\upsilon) \quad (2a)$$

$$c_{Cold}(\upsilon) = FFT(I_{Cold}) = r(\upsilon)B_{Cold}(\upsilon) - r(\upsilon)B_{Instr}(\upsilon) \quad (2b)$$

Therefore, from Equation (2b) the complex response is:

$$r(\upsilon) = \frac{FFT(I_{Cold})}{B_{Cold}(\upsilon) - B_{Instr}(\upsilon)} \quad (3)$$

For all calculations, the Planck function $B_{Cold}(\upsilon)$ is converted to a complex array with imaginary part $\equiv 0$.

Finally, from Equations (2a) and (2b) we obtain two equivalent expressions for $B_{Targ}(\upsilon)$ = the spectral radiance of the target:

$$B_{Targ}(\upsilon) = \frac{FFT(I_{Targ})}{r(\upsilon)} - B_{Instr}(\upsilon) \quad (4)$$

$$B_{Targ}(\upsilon) = B_{Cold}(\upsilon) + \frac{FFT(I_{Targ} - I_{Cold})}{r(\upsilon)} \quad (5)$$

Equation (5) is actually used to calculate the target radiance, which we designate as the "complex" radiance. Only the real part of $B_{Targ}(\upsilon)$ is utilized. The imaginary part is ≈ 0 and consists only of noise.

If phase correction of the target interferogram, I_{Targ}, is desired, then a scalar instrument spectral response, $R(\upsilon)$, is used instead of the complex response, $r(\upsilon)$ (Equation (3)).

$$R(\upsilon) = \frac{FFT(I_{Cold})_{SYM}}{B_{Cold}(\upsilon) - B_{Instr}(\upsilon)} \tag{6}$$

The spectral radiance of the target, $B_{Targ}(\upsilon)$, which we designate as the "scalar" radiance, then becomes:

$$B_{Targ}(\upsilon) = B_{Cold}(\upsilon) + \frac{FFT(I_{Targ} - I_{Cold})_{SYM}}{R(\upsilon)} \tag{7}$$

Focal Planes 3 and 4:

The mid-infrared Focal Planes 3 and 4 are calibrated by periodically viewing both a cold target (deep space at 2.7K) and a warm target (a shutter consisting of a mirror reflecting the interior of the instrument at ~ 170.0K). Two calibration targets are required because the temperature of FP3 and FP4 is ~ 78K, whereas the instrument temperature, T_{Instr}, is ~ 170.0K. Equations (8a) – (8c) express the linear relationship between the uncalibrated complex spectra and the target radiances:

$$c_{Targ}(\upsilon) = FFT(I_{Targ}) = r(\upsilon)B_{Targ}(\upsilon) - r(\upsilon)B_{Instr}(\upsilon)e^{i\theta} \tag{8a}$$

$$c_{Cold}(\upsilon) = FFT(I_{Cold}) = r(\upsilon)B_{Cold}(\upsilon) - r(\upsilon)B_{Instr}(\upsilon)e^{i\theta} \tag{8b}$$

$$c_{Warm}(\upsilon) = FFT(I_{Warm}) = r(\upsilon)B_{Warm}(\upsilon) - r(\upsilon)B_{Instr}(\upsilon)e^{i\theta} \tag{8c}$$

Solve for $r(\upsilon)$ = the complex instrument spectral response. From Equations (8b) and (8c) we obtain:

$$c_{Warm}(\upsilon) - c_{Cold}(\upsilon) = FFT(I_{Warm}) - FFT(I_{Cold}) = r(\upsilon)\left[B_{Warm}(\upsilon) - B_{Cold}(\upsilon)\right] \tag{9}$$

$$r(\upsilon) = \frac{FFT(I_{Warm} - I_{Cold})}{B_{Warm}(\upsilon) - B_{Cold}(\upsilon)} \tag{10}$$

since

$$FFT(I_{Warm}) - FFT(I_{Cold}) = FFT(I_{Warm} - I_{Cold}) \tag{11}$$

For all calculations, the Planck functions $B_{Warm}(\upsilon)$ and $B_{Cold}(\upsilon)$ are converted to complex arrays with imaginary parts ≡ 0.

Solve for I_{Instr} = the interferogram due to instrument radiance (self-emission). From Equations (8b) and (8c) we obtain:

$$r(\upsilon)B_{Instr}(\upsilon)e^{i\theta} = \frac{c_{Warm}(\upsilon)B_{Cold}(\upsilon) - c_{Cold}(\upsilon)B_{Warm}(\upsilon)}{B_{Warm}(\upsilon) - B_{Cold}(\upsilon)} \tag{12}$$

The total target interferogram, I_{Targ}, may be considered to be the sum of interferograms due to the target radiance, $B_{Targ}(\upsilon)$, and the instrument radiance (self-emission), $B_{Instr}(\upsilon)$. Applying an inverse FFT on Equation (8a) we obtain:

$$I_{Targ} = FFT^{-1}[c_{Targ}(\upsilon)] = FFT^{-1}[r(\upsilon)B_{Targ}(\upsilon)] - FFT^{-1}[r(\upsilon)B_{Instr}(\upsilon)e^{i\theta}] \tag{13}$$

Equation (13) may be rewritten as:

$$I_{Targ} = FFT^{-1}[r(\upsilon)B_{Targ}(\upsilon)] - I_{Instr} \tag{14}$$

where:

$$I_{Instr} = FFT^{-1}[r(\upsilon)B_{Instr}(\upsilon)e^{i\theta}] \tag{15}$$

From Equations (12) and (15) we therefore obtain:

$$I_{Instr} = FFT^{-1}\left[\frac{FFT(I_{Warm})B_{Cold}(\upsilon) - FFT(I_{Cold})B_{Warm}(\upsilon)}{B_{Warm}(\upsilon) - B_{Cold}(\upsilon)}\right] \tag{16}$$

Finally, using Equation (14) we solve for $B_{Targ}(\upsilon)$ = the spectral radiance of the target, which we designate as the "complex" radiance:

$$FFT^{-1}[r(\upsilon)B_{Targ}(\upsilon)] = I_{Targ} + I_{Instr} \tag{17}$$

$$B_{Targ}(\upsilon) = \frac{FFT(I_{Targ} + I_{Instr})}{r(\upsilon)} \tag{18}$$

Only the real parts of $B_{Targ}(\upsilon)$ and I_{Instr} are now used. The imaginary parts are ≈ 0 and consist only of noise.

If phase correction of the target interferogram, I_{Targ}, is desired, then a scalar instrument spectral response, $R(\upsilon)$, is used instead of the complex response, $r(\upsilon)$ (Equation (10)).

$$R(\upsilon) = \frac{FFT(I_{Warm} - I_{Cold})_{SYM}}{B_{Warm}(\upsilon) - B_{Cold}(\upsilon)} \tag{19}$$

The spectral radiance of the target, $B_{Targ}(\upsilon)$, which we designate as the "scalar" radiance, then becomes:

$$B_{Targ}(\upsilon) = \frac{FFT(I_{Targ} + I_{Instr})_{SYM}}{R(\upsilon)} \tag{20}$$

We use Equation (14) in its complex form to calculate a synthetic CIRS interferogram, I_{Targ}, for some target of interest, $B_{Targ}(\upsilon)$, e.g. Jupiter. $r(\upsilon)$ and I_{Instr} have been calculated from both flight and ground test data (May 2-3, 1997) and are available in text files as complex and real arrays, respectively. For the case of Jupiter, $B_{Targ}(\upsilon)$ = a synthetic Jupiter spectrum in radiance units (W cm^{-2} $ster^{-1}$/cm^{-1}). $B_{Targ}(\upsilon)$, $r(\upsilon)$, and I_{Instr} are substituted into Equation (14) to

calculate I_{Targ}. $B_{Targ}(\upsilon)$ and I_{Instr} are first converted to complex arrays, with imaginary parts $\equiv$ 0, for the calculation of I_{Targ}. The desired synthetic CIRS interferogram is then the real part of I_{Targ}; the imaginary part is expected to be ≈ 0 and consist only of noise.