



PROCESSING THE HASI MEASUREMENTS

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ABSTRACT

Huygens is an atmospheric probe that will descend on the surface of Titan. It contains an Atmospheric Structure Instrument (HASI). During the descent the instrument measures acceleration, total pressure and temperature. The atmospheric structure will be calculated from these measurements. The conditions in the atmosphere of Titan are not favourable for simple physical models. The characteristic thermodynamical quantities are estimated to vary up to about 10% from ideal ones. Hence it is recommended that the equations that will be used for calculations should be checked for invalid assumptions.

INTRODUCTION

Cassini is a joint Saturn mission of NASA and ESA dedicated to the exploration of the Saturnian system /1/. ESA is responsible for the Huygens probe which will descend to the surface of Titan 27 Nov 2004. The atmosphere of Titan is predominantly composed of Nitrogen, just like on Earth. The single most uncertain factor is the amount of Argon in the atmosphere, the estimates of which vary from none to 10 % mole fraction /2/. The radius of Titan is 2575 km, comparable to the Galilean satellites but, unlike those, it has a surface pressure of about 1.5 times the pressure on Earth, which makes Titan quite unique among known moons of our solar system. Fast zonal winds are expected to be seen, but their magnitude and direction are uncertain /3/. This uncertainty has implications for the probe descent trajectory.

HASI (Huygens Atmospheric Structure Instrument) will measure the vertical structure of the atmosphere of Titan. HASI consists of total pressure and temperature sensors and a triaxial accelerometer, which together give the density, temperature and ambient pressure profiles, as well as some information about winds and turbulence. At altitudes between 1200 and 180 km, the probe brakes with its thermal shield and the primary information about the composition of the atmosphere is obtained from the accelerometer measurements. In the stratosphere, below 170 km, the probe deploys a parachute, and the pressure and temperature data are measured directly. In the case that the surface of Titan is not too hard and the probe survives the impact, HASI will conduct some additional measurements on the surface. The Finnish Meteorological Institute produces the pressure measuring unit. Feasibility studies pointed towards a Pitot-type total pressure probe. This was then enhanced with a modified low-mass Kiel tube, which made the probe insensitive to the change in flow inclination angle up to 45°/4/. The actual probe is installed at the end of a deployable boom and has an inlet to the electronics box which also contains eight pressure sensors.

In an ideal case, all of the vertical profiles can be determined from the accelerometer data only. When the initial velocity, mass and the drag coefficient of the probe are known, the density of the gas and the velocity of the probe at a certain moment can be integrated. Substituting the density profile in the equation of hydrostatic equilibrium approximation the pressure can be obtained. The temperature is then obtained from the ideal gas law. In reality, however, there must be redundancy to prevent the cumulative effect of minor errors in the iterative process. The high-frequency ultrastable oscillator (USO) is in contact with the orbiter, and a Doppler measurement gives one velocity component during the whole descent. In a realistic case, two sets of equations have to be solved simultaneously. One of these is the basic set of hydrodynamics. These must be modified to take into account the special conditions on Titan. Another set describes the interaction of the probe in a flow field. The most important equations are those

which bind dynamic variables together with static ones. The effects of the Mach and Reynolds numbers, and the angle of attack on the drag coefficient must also be considered.

FLUID EQUATIONS

Real gases differ from ideal gases at low temperatures and high densities, where the intermolecular forces become significant. The corrective term in the ideal gas equation of state is the virial expansion, here noted as Z :

$$p = cRT(1 + Z(c, T)) \quad (1)$$

where p is the pressure (N/m^2), $c = \rho/m$ is the concentration (mol/m^3), ρ is the density and m the mean molecular mass, $R = 8.3143 \text{ Jmol}^{-1}\text{K}^{-1}$ is the universal gas constant, and T is the temperature (K). This notation makes Z a pure number that directly gives the deviation from ideal gas. Because Nitrogen is the dominant gas at the atmosphere of Titan, Z_N is the most important term when determining the total Z . Assuming an Ar/N_2 mole fraction of 0.1 and the most likely conditions on the surface of Titan, $T = 94 \text{ K}$, $\rho = 5.3 \text{ kgm}^{-3}$ [5], the Z values can be calculated from the functional forms for Nitrogen [6] and Argon [7], respectively. The difference in partial pressure is -2.8% for Nitrogen and -0.3% for Argon. Other constituents have even less effect on the total Z .

The usual derivations of the energy equation rely on the ideal gas law and a calorically perfect, i.e. constant heat capacity, gas. A different approach is used here. Starting with the fundamental equation for the differential of the internal energy E :

$$dE = TdS - pdV \quad (2)$$

where S is the entropy and V the volume of the system, it is straightforward to derive the equation for the temperature in an adiabatic process with no external heating or forces:

$$\frac{dT}{dt} = \frac{\alpha_p T}{c_V \kappa_T m c^2} \frac{dc}{dt} \quad (3)$$

where the thermodynamical responses are the volume expansivity α_p :

$$\alpha_p = -\frac{1}{c} \left(\frac{\partial c}{\partial T} \right)_p = \frac{1}{T} \left[1 + \frac{T}{1+Z} \left(\frac{\partial Z}{\partial T} \right)_c \right] \bigg/ \left[1 + \frac{c}{1+Z} \left(\frac{\partial Z}{\partial c} \right)_T \right] \quad (4)$$

the isothermal compressibility κ_T :

$$\kappa_T = \frac{1}{c} \left(\frac{\partial c}{\partial p} \right)_T = \frac{1}{p} \left(1 + \frac{c}{1+Z} \left(\frac{\partial Z}{\partial c} \right)_T \right)^{-1} \quad (5)$$

and the specific isochoric heat capacity c_V :

$$c_V(c, T) = c_V^0(T) + \frac{R}{m} T^2 \alpha_p \left(\frac{\partial Z}{\partial T} \right)_c \quad (6)$$

where c_V^0 is the ideal gas heat capacity. For the parameter range relevant to the measurements, it can be written for Nitrogen as

$$c_V^0 = \frac{R}{m} \left[\frac{5}{2} + \frac{x^2 e^x}{(e^x - 1)^2} \right] \quad \text{with} \quad x = \frac{T_0}{T} \quad (7)$$

where $T_0 = 3353$ K is the molecular vibration temperature [6]. Other gases can be modelled with the ideal gas heat capacity. Calculated using the Z function of Nitrogen, the deviations from the ideal responses at the surface conditions are for α_p 8.6%, for κ_T 2.9% and for c_v 2.3%.

INTERACTION EQUATIONS

Instead of the standard forms, the equations for an object in a fluid field are written here in their basic forms, avoiding assumptions as much as possible. A moving object in a fluid experiences a drag force F

$$F = -\frac{1}{2} \rho v v S C_D \quad (8)$$

where v is the velocity of the object relative to the fluid, S is a suitably chosen reference area, usually the cross section of the object, and C_D is the drag coefficient. The drag coefficient is not a constant, although it may vary only little over certain parameter ranges. The parameters of the coefficient are the two attack angles ϕ and θ , the free-stream Mach number M_∞

$$M_\infty = \frac{v_\infty}{c_s} \quad (9)$$

$$c_s = \frac{1}{\sqrt{mc\kappa_s}} \quad (10)$$

$$\kappa_s = \frac{\kappa_T c_v}{c_v + T(\alpha_p)^2 / mc\kappa_T} \quad (11)$$

where c_s is the speed of sound in the fluid and κ_s is the adiabatic compressibility, and the Reynolds number R_e

$$R_e = \frac{mc}{\eta} sv \quad (12)$$

where $\eta = \eta(c, T)$ is the dynamic viscosity and s is a relevant length scale. Most pronounced effects of these parameters reside in regions where the type of flow changes between subsonic and supersonic or between laminar and turbulent. The functional form of C_D has to be determined either from wind tunnel experiments or with computer simulations.

A pitot pressure probe measures the total pressure of the flow field at a stagnation point, i.e. in a point of flow field where the velocity of the field is zero. In reality the probe is a pressure gauge connected to a tube with its mouth opening against the flow. The relation between the stagnation point pressure p and the ambient pressure p_∞ is approximately

$$p_\infty = p \left(1 + \frac{v^2}{2c_p T_\infty} \right)^{-mc_p/R(1+Z)} \quad (13)$$

where c_p is the isobaric heat capacity

$$c_p = c_v + \frac{T\alpha_p^2}{mc\kappa_T} \quad (14)$$

The free-stream temperature is used instead of the stagnation point temperature, reflecting the instrument configuration.

CONCLUSIONS

Different simplifications of thermodynamical models introduce errors that are typically in the range of a couple of percent. Because one model can contain several such sources of error, the cumulative error of each equation must be determined. The use of approximations related to the dynamics of the flow field are not satisfactory at all, since their validity cannot be determined before the measurements have been processed, and algorithms that use these assumptions will produce results that are in agreement with the assumptions, thus preventing their validation.

The gases in the atmosphere of Titan deviate from ideal and calorically perfect ones near the surface, and from thermally perfect and nonreacting ones during the entry phase. The quantities that are calculated iteratively from the measurement data must take into account these deviations to prevent the accumulation of error. When the drag coefficient is determined, the phenomena in the flow field during the supersonic entry must be modelled correctly to avoid a large initial error in the entry velocity for the second phase of the descent.

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